Proceedings of the 2005 IEEE International Conference on Fuzzy Systems,
pp.347-353, Reno, USA, May 2005

Embedded Type-2 FLC for the Speed Control of Marine and Traction Diesel Engines

Christopher Lynch, Hani Hagras, Victor Callaghan
Department of Computer Science, University of Essex
Wivenhoe Park, Colchester, CO4 3SQ, UK

***Abstract*- Marine and Traction propulsion systems operate in highly dynamic and uncertain control environments. The current speed controllers for marine/traction propulsion systems are based on PID and type-1 Fuzzy Logic Controllers (FLCs) which cannot fully handle the uncertainties associated with such dynamic environments. A type-2 FLC can handle such uncertainties to produce a better performance. However, type-2 FLCs have a computational overhead associated with the iterative type-reduction process which can reduce the FLC real-time performance as embedded controllers generally have limited computational and memory capabilities. In this paper, we will introduce a real-time type-2 FLC that is suited for embedded controllers operating in marine/traction propulsion system. We have conducted numerous experiments where the embedded type-2 FLCs displayed a robust response, dealing with the uncertainties in real-time, outperforming PID and type-1 FLCs, whilst using smaller rule bases.**

I. INTRODUCTION

Marine and Traction diesel propulsion systems are huge engines which require accurate speed control. Effective control becomes even more vital when multiple engines are required to operate in parallel i.e. when both engines share the load and must react together to overcome load changes. Fig.1 shows a typical application engine, the RK280 a 20 cylinder diesel engine. It has a mass of 46,000kg and power output of 9000kW. A typical application for this engine would be a Fast Ferry, due to its high power to weight ratio. Obviously speed control of an engine of this size is critically important as engines of this power can accelerate at a rate of 200% per sec^2.

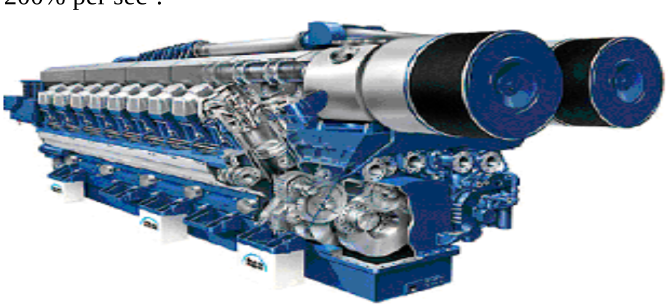


Fig. 1. 20 Cylinder MAN B&W RK280, 9000 kW Engine

Marine/traction propulsion systems operate in highly dynamic and uncertain environments, experiencing vast changes in ambient temperature, altitude, fuel, humidity and load. Choosing an appropriate speed control mechanism to model and handle these uncertainties is of vital importance.

Traditionally various forms of PID controllers has been used for speed control in marine/traction propulsion systems due to their simplicity. A great deal of research has also gone into providing more robust and optimal PID controllers through various tuning techniques including Zeigler Nichols, Cohen-Coon and the Chien, Hrones and Reswick (CHR) methods [1], although in practise these techniques usually only provide a starting point for further manual tuning by experienced engineers.

Alternatively AI researchers have shown that Fuzzy Logic and Fuzzy PID controllers can provide improved control and robustness over traditional PID [1], [2]. As a result FLCs have found use in the speed control of various marine/traction propulsion systems [3], [4]. However, there are many sources of uncertainty facing the FLC for marine/traction propulsion systems; we list some of them as follows:

- Uncertainties in inputs to the FLC which translates to uncertainties in the antecedents membership functions as the sensors measurements are affected by noise from the vibration associated with these huge machines, as well as external sources of noise. The sensors are also affected by poor circuit design, calibration errors as well as environmental effects (i.e. their characteristics are changed by the temperature, humidity, etc.).
- Uncertainties in control outputs which translate to uncertainties in the output membership functions of the FLC. Such uncertainties can result from the change of the actuators characteristics due to wear or tear or due to the environmental changes. For example the calorific value of fuel can vary with temperature for a ferry travelling between different geographical locations which will have a direct effect on combustion.
- Uncertainties associated with sudden changes in load which causes problems for an FLC which was fine tuned for a certain load.
- Linguistic uncertainties as the meaning of words that are used in the antecedents and consequents linguistic labels can be uncertain, as words mean different things to different people [5]. In addition, experts do not always agree and they often provide different consequents for the same antecedents. A survey of experts will usually lead to a histogram of possibilities for the consequent of a rule, this histogram represents the uncertainty about the consequent of a rule [6].

All of these uncertainties translate into uncertainties about fuzzy set membership functions [5]. These uncertainties cause difficulty in determining the exact and precise

 This work is funded by MAN B& W Diesel Ltd

antecedent and consequent fuzzy membership functions.
Type-1 FLCs have the problem that they cannot fully handle or accommodate the linguistic and numerical uncertainties associated with changing and dynamic environments, as they use precise type-1 fuzzy sets. Type-1 fuzzy sets handle the uncertainties associated with the FLC inputs and outputs by using *precise and crisp* membership functions that the user believes capture the uncertainties [7]. Once the type-1 membership functions have been chosen, all the uncertainty disappears, because type-1 membership functions are totally precise [7]. The uncertainties associated with the changing and dynamic environments of the marine/traction propulsion systems create problems in determining the exact (and precise) antecedents and consequents membership functions during FLC design. Moreover, the designed type-1 fuzzy sets can be sub-optimal under specific environment and load conditions, however because of the load and environment changes and the associated uncertainties the chosen type-1 fuzzy sets might not be appropriate anymore. This can cause degradation in the FLC performance and we might end up wasting time in frequently redesigning or tuning the type-1 FLC so that it can deal with the various uncertainties faced in dynamic environments [8].

A type-2 fuzzy set is characterised by a fuzzy membership function, i.e. the membership value (or membership grade) for each element of this set is a fuzzy set in [0,1], unlike a type-1 fuzzy set where the membership grade is a crisp number in [0,1] [9]. The membership functions of type-2 fuzzy sets are three dimensional and include a footprint of uncertainty, it is the new third-dimension of type-2 fuzzy sets and the footprint of uncertainty that provide additional degrees of freedom that make it possible to directly model and handle uncertainties [5], [7]. The type-2 fuzzy sets are useful where it is difficult to determine the exact and precise membership functions [9] (which is our case). The interval type-2 FLC presented in this paper uses interval type-2 fuzzy sets to represent the inputs and outputs of the FLC.

However, interval type-2 FLC involves a computational overhead associated with the computation of the type-reduced fuzzy ???? using the iterative *Karnik-Mendel* method [7], which is directly proportional to the number of rules. This computational overhead translates to a slower controller response which will reduce the robustness as well as limiting and disturbing the real time operation of the type-2 FLC, especially for a large rule base. Moreover, embedded controllers are generally run on embedded computer boards with limited computational and memory capabilities. Thus, the type-2 computational overhead will limit the applications of type-2 FLCs in industrial embedded controllers.

Wu and Mendel [10] introduced a method to approximate the type-reduced set by the inner and outer bound sets. This method, based on minimising a risk function to achieve similar outputs to the type-reduced outputs, needs training data in order to create an FLC. However, this data is difficult to acquire in our application, as the system will encounter a lot of unforeseen circumstances. In this paper we will use the *Wu-Mendel* method as an approximation to the iterative *Karnik-Mendel* method. We found, through extensive set of control experiments, that without acquiring data and minimising the risk function, the difference between the output of the two methods is very small. This makes the *Wu-Mendel* output a good approximation for the type-reduced sets, which will enable real time operation for embedded controllers operating in marine/traction systems.

There has been work on real-time type-2 FLCs for mobile robots [8]. There have been also work on type-2 FLCs for simulated industrial processes [11], [12]. However, to the authors knowledge this will be the first work investigating the application of real-time embedded type-2 FLC to a heavy industrial application, in this case a marine/traction system..

In section II we will review the hardware and software that will be used in the different stages of the project. Section III introduces the interval type-2 FLC. Section IV discusses the controllers design. Section V presents our experiments and results followed by the conclusions in section VI.

II. THE TEST-BEDS

This work is funded by MAN B&W Diesel Ltd which is one of the biggest manufacturers of marine and traction diesel propulsion systems and engines in the world. Due to the size and cost of the engines it was not possible to experiment on the engines themselves, rather our work utilises. three graduated platforms through which we can verify our system and control algorithms. These platforms are real systems, not software simulations, so as to be as close to the real conditions as possible. The first platform is a brushless DC motor used to both determine the appropriate hardware and software needed by the embedded controller and to establish the benefits of type-2 FLCs for real-time control. The second platform is a mechanical engine simulator shown in Fig.2. These huge simulators, which provide real physical load conditions, are used in industry to verify the engine speed controllers operation. before deployment on the target engine.. The third platform is an actual marine/traction diesel engine.. The work reported in this paper reports on results from the first platform..

Fig. 2. The Mechanical Simulator

The chosen hardware for the embedded controller was based on the Texas Instruments TMS320F2812 150MHz

DSP programmed in ANSI C. We used it to control a 3 Phase brushless dc motor via a 50W power inverter. Hall-Effect sensors and a 500 line quadrature encoder where used for commutation and velocity measurements respectively. Load on the motor was applied via an Eddy Current brake, consisting of 10.5cm aluminium disc coupled to the brushless motor. Magnetic flux was applied utilising several permanent magnets at fixed distances from the disc representing loads of 25%, 50%, 75% and 100%. Feedback from the DSP was achieved via a Controller Area Network (CAN serial communication protocol) bus using Vectors automotive "CAN analyzer" Software. This testbed is shown in Fig. 3.

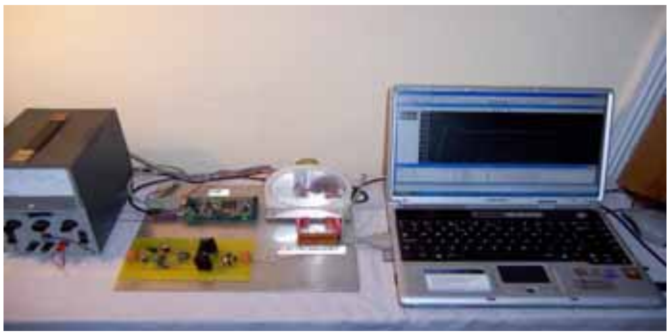


Fig. 3. The test bed.

III. INTERVAL TYPE-2 FLC

The interval type-2 FLC is depicted in Fig.4 and consists of a Fuzzifier, Inference Engine, Rule-Base, Type Reducer and Defuzzifier. We now discuss briefly the operation of each component of the FLC,with emphasis on type-reduction.

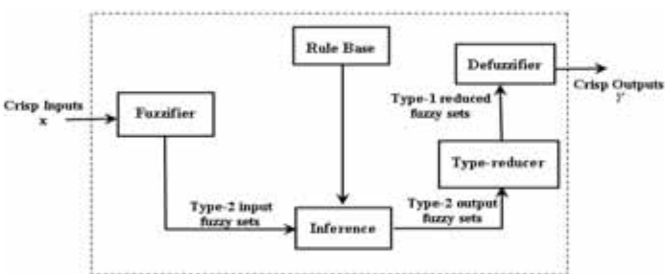


Fig. 4. Type-2 FLC.

A. *Fuzzifier*

The fuzzifier maps a crisp input to a Type-2 fuzzy set [7]. Singleton fuzzification was chosen due to its low computational burden. Using singleton fuzzification two membership values are calculated for each fuzzy set i.e. the lower ($\underline{\mu}$) and upper ($\overline{\mu}$) membership values.

B. *Inference Engine and Rule Base*

The inference engine combines rules and gives a mapping from input type-2 sets to output type-2 sets. The firing strengths are calculated for all the fired rules. The firing strength f^i of the i^{th} rule (($i=1\ldots M$) where M is the number of rules) is an interval type-1 set determined by its left most point $\underline{f}^i(\mathbf{x'})$ and its right most point $\overline{f}^i(\mathbf{x'})$ which are calculated as follows:

$$\underline{f}^i(\mathbf{x'}) = \underline{\mu}_{\tilde{F}_1^i}(x_1') * \ldots\ldots * \underline{\mu}_{\tilde{F}_p^i}(x_p') \qquad (1)$$

$$\overline{f}^i(\mathbf{x'}) = \overline{\mu}_{\tilde{F}_1^i}(x_1') * \ldots\ldots * \overline{\mu}_{\tilde{F}_p^i}(x_p') \qquad (2)$$

Where p is the number antecedents for the i^{th} that rule and '*' denotes the product operation.

C. *Type Reduction*

Type-reduction translates the type-2 output sets of the inference engine to a type-1 set; this is called a "type-reduced set" [7]. These type-reduced sets are then defuzzified to obtain a crisp output used for control.

We will use the "centre-of-sets" type reduction, as it has reasonable computational complexity (lying between computationally expensive centroid type-reduction and the simple height and modified height type-reductions which have problems when only one rule fires) [7]. The type-reduced set using the "centre-of-sets" type-reduction is expressed below [7],

$$Y_{\cos}(x) = [y_l(x), y_r(x)] = \int_{y^i \in [y_l^i, y_r^i]} \int_{f^i \in [\underline{f}^i, \overline{f}^i]} 1 \Big/ \frac{\sum_{i=1}^{M} f^i y^i}{\sum_{i=1}^{M} f^i} \qquad (3)$$

Where the type reduced set $Y_{\cos}(x)$ is an interval set defined by its left and right most points y_l (x), y_r (x). y^i corresponds to the centroid of the type-2 consequent of the fired i^{th} rule which is determined by its left most point y_l^i and its right most point y_r^i.

Assuming the consequent centroids of each firing rule have been pre-calculated, the next step in type reduction is to implement the *Karnik-Mendel* iterative procedure [7], [13]. This procedure has been proven to calculate both y_l (x), y_r (x) in no more than *2M* iterations (where *M* is the number of rules) [7], [13]. As explained above this procedure is a computationally expensive algorithm and not suited for real-time operation in embedded controllers unless the rule base is particularly small [8].

Alternatively the *Wu-Mendel* Uncertainty Bounds [10] provides mathematical formulas for inner and outer bound sets which can be used to approximate the type-reduced set and can further be used to directly derive the defuzzified output [10].Using this method will avoid the iterative *Karnik-Mendel* procedure which creates a computational bottleneck for the type-2 FLC embedded controller. In this work we have conducted extensive experiments comparing the defuzzified output of the *Karnik-Mendel* iterative procedure and the defuzzified output of the *Wu-Mendel* defuzzified output. As we will shown in the experiments section, we found that the difference between the two outputs is very small for the control application. This justifies the use of the

Wu-Mendel method as a valid approximation to *Karnik-Mendel* iterative procedure, even without collecting training data and minimising a risk function as in [10]. This data is difficult to acquire in our system, as it operate mostly in unknown environments and encounters a lot of unforeseen situations.

Equations (4) and (5) define the inner bound while (6) and (7) define the outer bound for the centre-of-sets type reduction method [10].

$$\underline{y}_l(x) = \min\left\{ \frac{\sum_{i=1}^{M} \underline{f}^i y_l^i}{\sum_{i=1}^{M} \underline{f}^i}, \frac{\sum_{i=1}^{M} \overline{f}^i y_l^i}{\sum_{i=1}^{M} \overline{f}^i} \right\} \quad (4)$$

$$\underline{y}_r(x) = \max\left\{ \frac{\sum_{i=1}^{M} \underline{f}^i y_r^i}{\sum_{i=1}^{M} \underline{f}^i}, \frac{\sum_{i=1}^{M} \overline{f}^i y_r^i}{\sum_{i=1}^{M} \overline{f}^i} \right\} \quad (5)$$

$$\underline{y}_l(x) = \overline{y}_l(x) - \left[\frac{\sum_{i=1}^{M} (\overline{f}^i - \underline{f}^i)}{\sum_{i=1}^{M} \overline{f}^i \sum_{i=1}^{M} \underline{f}^i} * \frac{\sum_{i=1}^{M} \underline{f}^i (y_l^i - y_l^1) \sum_{i=1}^{M} \overline{f}^i (y_l^M - y_l^i)}{\sum_{i=1}^{M} \underline{f}^i (y_l^i - y_l^1) + \sum_{i=1}^{M} \overline{f}^i (y_l^M - y_l^i)} \right] \quad (6)$$

$$\overline{y}_r(x) = \underline{y}_r(x) + \left[\frac{\sum_{i=1}^{M} (\overline{f}^i - \underline{f}^i)}{\sum_{i=1}^{M} \overline{f}^i \sum_{i=1}^{M} \underline{f}^i} * \frac{\sum_{i=1}^{M} \overline{f}^i (y_r^i - y_r^1) \sum_{i=1}^{M} \underline{f}^i (y_r^M - y_r^i)}{\sum_{i=1}^{M} \overline{f}^i (y_r^i - y_r^1) + \sum_{i=1}^{M} \underline{f}^i (y_r^M - y_r^i)} \right] \quad (7)$$

D. *Defuzzification*

Defuzzification for the Karnik *Mendel iterative procedure* is computed as follows:

$$\frac{y_l(x) + y_r(x)}{2} \quad (8)$$

Defuzzification for the *Wu Mendel Uncertainty Bounds* method is defined below:

$$\frac{\frac{\underline{y}_l(x) + \overline{y}_l(x)}{2} + \frac{\underline{y}_r(x) + \overline{y}_r(x)}{2}}{2} \quad (9)$$

Equations (8) and (9) both produce a crisp output to be used to control our process.

IV. THE CONTROLLERS DESIGN

Four speed controllers were implemented each of which has two antecedents termed as:

- Error 'e', difference between the process set-point and current process state.
- Rate of change of error 'd' i.e. $\frac{de}{dt}$.

They have a single consequent corresponding to the speed output. A brief description of each controller is given below:

A. *The PID Controller*

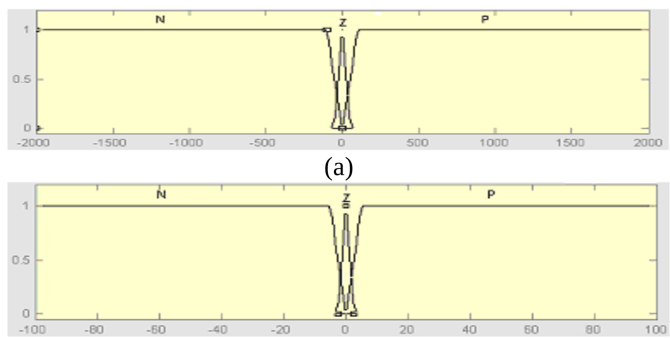
The PID controller is the most widely used controller in marine/propulsion traction systems. The PID equation in its discrete form for use in the digital embedded controllers can be expressed as:

$$u_n = Kp(e_n + \frac{1}{Ti}\sum_{z=1}^{n} e_z Ts + Td\frac{e_n - e_{n-1}}{Ts}) \quad (10)$$

Where n is the current sample, u_n is the manipulating variable, Kp is the proportional gain, T_i the integral time and T_d the derivative time and e is the error and T_s the sampling period in seconds. The proportional gain and derivative/integral times were initially chosen using the Zeigler Nichols frequency response method before further tuning for improved control.

B. *Type-1 FLC*

A 9 rule Mamdani Type-1 FLC was implemented. The membership functions for the antecedents are in Fig.5, where each antecedent is represented using three fuzzy sets which are *Negative 'N'*, *Zero 'Z'* and *Positive 'P'*.



(a)

(b)

Fig.5. Type-1 memberships functions for the antecedents (a) e, (b) d

The FLC consequent (the speed output) is represented by five fuzzy sets as shown in Fig.6 which are *Negative Large 'NL'*, *Negative Small 'NS'*, *Zero 'Z'*, *Positive Small 'PS'* and *Positive Large 'PL'* .

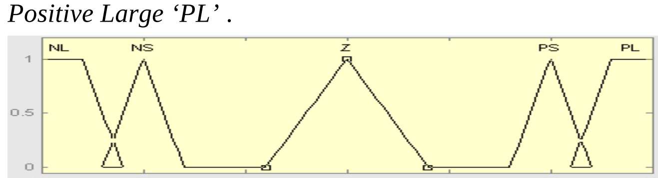


Fig.6. Type-1 memberships functions for the consequents.

The rule base of the Type-1 FLC is made up of nine rules as shown in Table 1(a).

C. *Type-2 FLC*

We tried two type-2 FLCs which are identical but using two different type-reduction methods. The first used the iterative *Karnik-Mendel* procedure which we term the 'Iterative type-2 FLC' and the other uses the *Wu-Mendel* uncertainty bounds and we term 'Bounds type-2 FLC' Both type-2 FLCs use two type-2 fuzzy sets to represent each input as shown in Fig. 7(a) and Fig. 7(b). The footprint of uncertainty has incorporated the numerical and linguistic uncertainties the motors will need to deal with. The rule base of two type-2 FLCs consists of 4 rules as shown in Table 1(b).

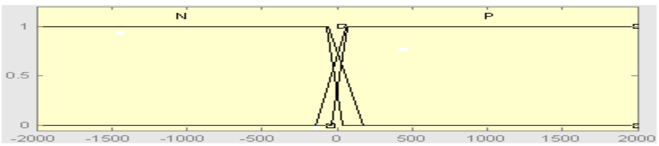
TABLE 1: (a) Type-1 FLC Rule Base (b) Type-2 FLC Rule Base

(a)

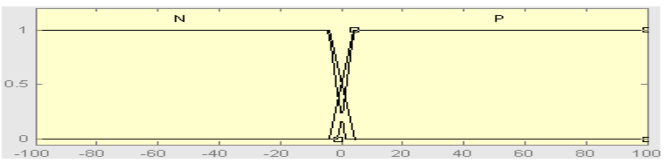
d \ e	**N**	**Z**	**P**
N	*NL*	*NS*	*PL*
Z	*NL*	*Z*	*PL*
P	*NL*	*PS*	*PL*

(b)

d \ e	**N**	**P**
N	*NL*	*PM*
P	*NS*	*PL*



(a)



(b)

Fig.7. Type-2 FLC antecedents (a) ‘*e*’, (b) ‘*d*’

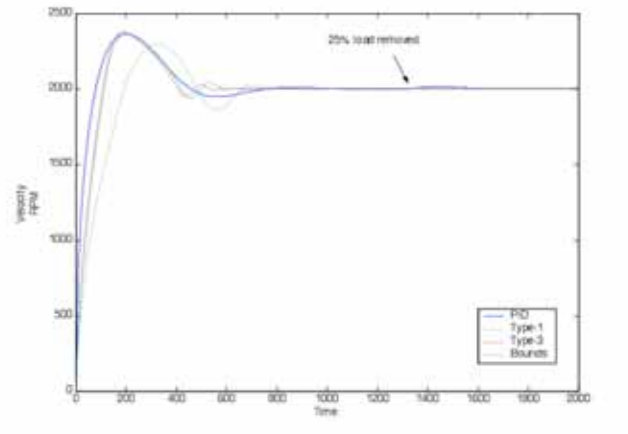
V. Experiments and Results

Each of the four controllers was coded in ANSI C and embedded in the DSP board. Each controller was tuned to give optimal performance on 25% (nominal load) of the total load, also an arbitrary set-point of 2000 rpm was chosen. In each experiment the controllers were allowed to reach set-point and stabilise with the 25% constant load. The experiments consisted of adding/removing load (indicated by an arrow in each figure) to mimic the change of load facing the marine/traction propulsion systems. In this industry it is necessary for the controller to be able to deal quickly with the uncertainties associated with a change of load, producing minimum overshoot/undershoot (as big overshoots can cause damage due to the large size and hight speeds of the machines).. Also, it is desirable not to retune or redesign the controller with each change of load or operating conditions.

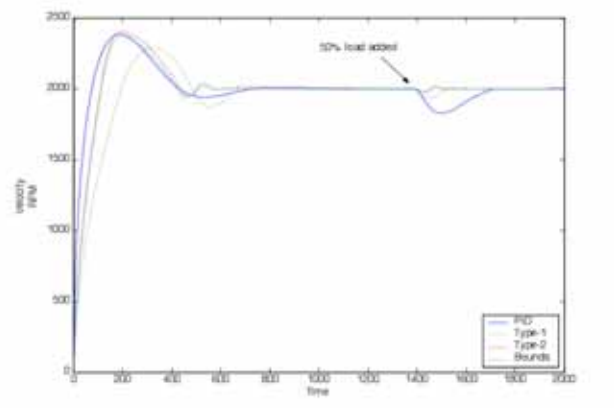
The first section of each graph corresponds to nominal operating load (note that the type-2 FLCs had produced a smooth response settling to the set-point with almost zero steady state error, which was faster than both the type-1 FLC and the PID controller). The type-2 FLCs used only 4 rules compared to the 9 rules used in the type-1 FLC. Note also that the type-2 FLCs produced a compromise beytween overshoot and rise time. Fig.8 (a) shows the first experiment which was to remove the 25% load. Each FLC was unaffected by the removal of the load and only the PID controller suffered a slight overshoot of 13 rpm before returning to set-point.

Fig.8 (b) shows the second experiment which involved adding a 50% load. Clearly, the PID performed the worst producing the largest undershoot and settling time. The type-1 FLC performed better whilst the type-2 FLCs were the fastest to recover, with only a slight disturbance.

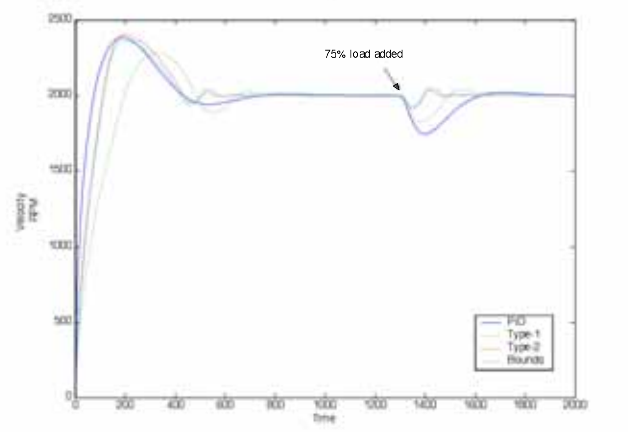
The experiment in Fig.8 (c) involved adding a 75% load. The PID had the largest undershoot but equivalent settling time to that of the type-1 FLC. As shown, the type-2 FLCs produced a very good performance with minimum settling time and undershoot, outperforming both the PID and type-1 FLC.



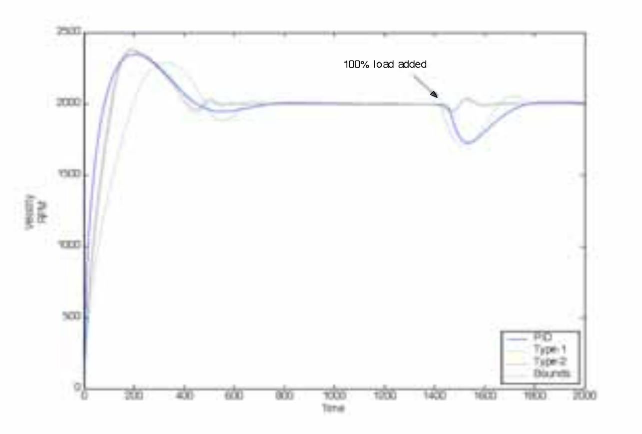
(a)



(b)



(c)



(d)
Fig.8. Experimental results of load changes

The experiment in Fig.8 (d) involved adding a 100% load. The PID and Type-1 controllers both performed equivalently, with similar undershoot and settling times. The type-2 FLCs outperformed both controllers, producing minimum disturbance.

It is clear that as the load increased (hence increasing the uncertainty) the type-2 FLCs handled these uncertainties to produce a very good control performance (ie fast recovery and minimal overshoot/undershoot, thus satisfying the requirements for the marine/traction propulsion industry) that out-performed the other controllers with the type-1 controller degrading to the equivalent performance of a PID controller as both controllers cannot handle the uncertainties.

Therefore, we can conclude that type-2 FLCs are more robust in the face of uncertainty than type-1 or PID controllers. These results are inline with the results reported in [8], [11], [12]. We also noticed that as more fuzzy sets are used to represent each input in the type-1 FLCs (hence more rules), the type-1 FLCs approaches the performance. This is because the type-2 fuzzy sets contain a large number of embedded type-1 fuzzy sets, and the type-2 FLC can be thought of as a collection of many different *embedded* type-1 FLCs [7] to deal with the different uncertainties.

Another very important result is that *Wu-Mendel* Uncertainty Bounds method produced very similar results to that of the *Karnik-Mendel* iterative procedure. When calculating the RMSE between the two controllers we had a very small error within 1% of set-point. This shows that we can use the *Wu-Mendel* Bounds method as a valid approximation for the type-reduction process. and thus as a useful and valid a means for real time control operations.

VI. CONCLUSIONS

In this paper, we presented the application of embedded type-2 FLC to marine/traction propulsion systems based on the first stage of the project involving the real time speed control of a brushless motor. We have shown that the *Wu-Mendel* Bounds methods can be used as a valid approximation for the type-reduced sets. This makes the application of type-2 FLC feasible and easier for embedded controllers. We have shown how the type-2 FLC can handle the uncertainties and produce a robust and smooth performance which outperforms the PID and Type-1 FLC which uses more rules.

For our future work we plan to install the system on an actual marine engine and assess its performance over time in real operational conditions, thus aiming at being the first industrial type-2 FLC applied to heavy engine control. Other work includes online adaptation of the type-2 Fuzzy Sets and rules with the aim of producing a self tuning type-2 engine governor.

ACKNOWLEDGMENT

We are pleased to acknowledge the funding support from MAN B&W Diesel Ltd. We are also pleased to acknowledge the great help of all the team of Regulator Europa and especially Kevin Moorey and Martin Birkin. Finally we acknowledge our colleagues in the IIEG (http://iieg.essex.ac.uk , particularly Martin Colley, whose support has been greatly appreciated.

REFERENCES

[1] K.Astrom, “PID Controllers: Theory, Design and Tuning”, ISA, 1995, ISBN:1-55617-516-7

[2] M. Golob, B. Tovornik, ”Real Time Fuzzy PID Controller Structures”, *Computational Intelligence and Applications*, World Scientific and Engineering Society Press, pp. 205-210,1999

[3] A. Amer, S. Eweda and M. Eweda, “Speed Control of Marine Diesel Engine Using Fuzzy Approach”, 12th International Conference on Computer Theory and Applications

[4] B.Bose, “Fuzzy Logic Based Intelligent Control of a Variable Speed Cage Machine Wind Generation System”, IEEE Transactions on Power Electronics, vol. 12, no. 1, January 1997

[5] J. Mendel, R. John, “Type-2 Fuzzy Sets Made Simple”, IEEE Transactions on Fuzzy Systems, vol. 10, no. 2, April 2002

[6] J. Mendel and H. Wu, "Uncertainty versus choice in rule-based fuzzy logic systems," Proceedings of IEEE International Conference on Fuzzy Systems, pp. 1336-1342, Honolulu, USA, 2002.

[7] J. Mendel, “Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions”, Upper Saddle River, NJ: Prentice-Hall, 2001

[8] H. Hagras, “A Hierarchical Type-2 Fuzzy Logic Control Architecture for Autonomous Mobile Robots”, IEEE Transactions on Fuzzy Systems, Vol.12,No.4,August 2004.

[9] Q. Liang, N. Karnik and J. Mendel, "Connection admission control in ATM networks using survey-Based type-2 fuzzy logic systems," IEEE Transactions on Systems, Man and Cybernetics Part C: Applications and Reviews, Vol. 30, pp. 329-339, August 2000.

[10] H. Wu, J. Mendel, “Uncertainty Bounds and their use in the Design of Interval Type-2 Fuzzy Logic Systems”, IEEE Transactions on Fuzzy Systems, Vol.10, No.5, Oct 2002.

[11]W. Tan, J. Lai, “Development of a Type-2 Fuzzy Proportional Controller”, *Proceedings of the 2004 IEEE International Conference on Fuzzy Systems*, Budapest, Hungary, July 2004.

[12] D. Wu, W. Tan,“A Type-2 Fuzzy Logic Controller for the Liquid Level Process”, *Proceedings of the 2004 IEEE International Conference on Fuzzy Systems*, Budapest, Hungary, July 2004

[13] Q. Liang, J. M. Mendel, “Interval type-2 fuzzy logic systems: Theory and design,” *IEEE Trans. Fuzzy Systems*, vol. 8, pp. 535–550, October 2000.